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Consistency Condition for the Pinch Technique Self-Energies at Two Loops

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ABSTRACT

A simple and testable necessary condition for the gauge independence of the Pinch Technique self-energies at two loops is discussed. It is then shown that, in the case of the Z and W self-energies, the condition is indeed satisfied by the Papavassiliou-Pilaftsis formulation.

The Pinch Technique (PT) is a convenient algorithm that automatically rearranges S-matrix elements of gauge theories into modified, gauge-independent self-energies, vertex, and box diagrams. In turn, the new corrections exhibit very desirable theoretical properties. For these reasons, the PT has been frequently employed in recent discussions of QCD and Electroweak Physics [1]. A temporary drawback is that the approach has been fully developed only at the one-loop level. Very recently, however, Papavassiliou and Pilaftsis (P-P) proposed a method to construct PT self energies at higher orders [2].

Calling $\widehat{\Pi}$ and Π the PT and R_ξ transverse self-energies, respectively, and focusing on the Z case, one has

$$\widehat{\Pi}^{ZZ}(s) = \Pi^{ZZ}(s) + (\Pi^{ZZ}(s))^P \ , \tag{1}$$

where the "pinch part" $(\Pi^{ZZ}(s))^P$ has the structure

$$(\Pi^{ZZ}(s))^P = c_1(s - M_0^2)V^P(s) + c_2(s - M_0^2)^2 B^P(s) - R^{ZZ}(s) \ . \tag{2}$$

In Eq.(2) the bare mass M_0 is assumed to be defined in a gauge invariant manner, tadpole contributions are included in $\Pi^{ZZ}(s)$, $V^P(s)$ and $B^P(s)$ are the pinch parts from vertex and box diagrams, respectively, and $R^{ZZ}(s)$ is a residual amplitude of $\mathcal{O}(g^4)$ proposed in Ref.[2]. It is discussed in detail later on at the $\mathcal{O}(g^4)$ level. Because of the limited knowledge currently available concerning multi-loop amplitudes in gauge theories, a general proof that $\widehat{\Pi}^{ZZ}(s)$ is gauge invariant in higher orders is not presently available. One of the aims of this report is to note that by judiciously restricting the domain of s to lie in the neighborhood of $\bar{s}$, the complex-valued position of the propagator's pole, one can obtain an expression for which the gauge independence can be tested on the basis of current knowledge. Specifically, we consider the neighborhood $|s - \bar{s}| \leq \mathcal{O}(g^2|\bar{s}|)$, which roughly includes the resonance region. Recalling that $\bar{s} - M_0^2 = \mathcal{O}(g^2)$, through $\mathcal{O}(g^4)$ Eqs.(1,2) become

$$\widehat{\Pi}^{ZZ}(s) = \Pi_1^{ZZ}(s) + c_1(s - M_0^2)V_1^P(s) + \Pi_2^{ZZ}(\bar{s}) - R_2^{ZZ}(\bar{s}) + \mathcal{O}(g^6) \ , \tag{3}$$

where the indices $i = 1, 2$ denote $\mathcal{O}(g^2)$ and $\mathcal{O}(g^4)$ contributions. Through $\mathcal{O}(g^4)$ the first two terms in the r.h.s. of Eq.(3) equal the one-loop PT self-energy $\widehat{\Pi}_1^{ZZ}(s)$, which is

$\xi-$independent . Its explicit expression [3] is :

$$\widehat{\Pi}_1^{ZZ}(s) = \Pi_1^{ZZ}(s)|_{\xi_i=1} - 4g^2c_w^2(s-M_0^2)I_{WW}(s) \ , \tag{4}$$

where c_w^2 is an abbreviation for $\cos\theta_w^2$, ξ_i $(i = W, Z, \gamma)$ are the R_ξ gauge parameters and

$$I_{ij}(q^2) = i\mu^{4-n}\int\frac{d^nk}{(2\pi)^n}\frac{1}{(k^2-M_i^2)\left[(k+q)^2-M_j^2\right]} \ . \tag{5}$$

On the other hand, $\Pi_2^{ZZ}(\bar{s})$ in Eq.(4) is expected to be gauge dependent. Although this amplitude is not fully known, its gauge-dependent part can be isolated by a simple argument. Recalling that the pole position $\bar{s} = M_0^2 + \Pi^{ZZ}(\bar{s}) + [\Pi^{\gamma Z}(\bar{s})]^2/[\bar{s} - \Pi^{\gamma\gamma}(\bar{s})]$ is gauge invariant, through $\mathcal{O}(g^4)$ we have

$$\Pi_2^{ZZ}(\bar{s}) = \bar{s} - M_0^2 - \widehat{\Pi}_1^{ZZ}(\bar{s}) + c_1(\bar{s}-M_0^2)V_1^P(\bar{s}) - [\Pi_1^{\gamma Z}(\bar{s})]^2/\bar{s} \ , \tag{6}$$

where $\widehat{\Pi}_1^{ZZ}(\bar{s})$ is defined in Eq.(4). The amplitude $c_1V_1^P(s)$ can be gleaned from Eqs.(12b,16d) of Ref.[3] :

$$c_1V_1^P(s) = -4g^2c_w^2\left[I_{WW}(s) + (\xi_W-1)\upsilon_W(s)/2\right] \ , \tag{7}$$

where $\upsilon_W(s)$ is a ξ_i-dependent function defined in Eqs.(2-6) of Ref.[4]. Recalling $\bar{s}-M_0^2 = \Pi_1^{ZZ}(\bar{s})|_{\xi_i=1} + \mathcal{O}(g^4)$, we have

$$c_1(\bar{s}-M_0^2)V_1^P(\bar{s}) = -4g^2c_w^2\Pi_1^{ZZ}(\bar{s})|_{\xi_i=1}\left[I_{WW}(\bar{s}) + (\xi_W-1)\upsilon_W(\bar{s})/2\right] \ . \tag{8}$$

On the other hand, using Eq.(8) of Ref.[4], one finds for general ξ_i

$$\begin{aligned}\frac{\left[\Pi_1^{\gamma Z}(\bar{s})\right]^2}{\bar{s}} &= \left[\Pi_1^{\gamma Z}(\bar{s})|_{\xi_i=1} + g^2s_wc_w(\xi_W-1)\bar{s}\upsilon_W(\bar{s})\right]^2/\bar{s} + \mathcal{O}(g^6)\\ &= [\Pi_1^{\gamma Z}(\bar{s})|_{\xi_i=1}]^2/\bar{s} + 2g^2s_wc_w(\xi_W-1)\upsilon_W(\bar{s})\Pi_1^{\gamma Z}(\bar{s})|_{\xi_i=1}\\ &\quad + g^4s_w^2c_w^2(\xi_W-1)^2\bar{s}\upsilon_W^2(\bar{s}) + \mathcal{O}(g^6) \ .\end{aligned} \tag{9}$$

Combining Eqs.(6,8,9) we obtain

$$\begin{aligned}\Pi_2^{ZZ}(\bar{s}) &= \Pi_2^{ZZ}(\bar{s})|_{\xi_i=1} - 2g^2(\xi_W-1)\upsilon_W(\bar{s})\left[c_w^2\Pi_1^{ZZ}(\bar{s})|_{\xi_i=1} + c_ws_w\Pi_1^{\gamma Z}(\bar{s})|_{\xi_i=1}\right]\\ &\quad - g^4s_w^2c_w^2(\xi_W-1)^2\bar{s}\upsilon_W^2(\bar{s}) \ .\end{aligned} \tag{10}$$

The terms proportional to $\xi_W - 1$ and $(\xi_W-1)^2$ in Eq.(10) represent the ξ_i-dependent parts of $\Pi_2^{ZZ}(\bar{s})$. It follows that, if the residual contributions $R_2^{ZZ}(\bar{s})$ are not included,

Eq.(3) is gauge-dependent in $\mathcal{O}(g^4)$. Next we evaluate $R_2^{ZZ}(\bar{s})$. Following the P-P method [2], R_2 is the contribution that must be added to the chain of R_ξ transverse one-loop self-energies and corresponding pinch parts (Fig.1(b-d)), in order to convert it into the chain of one-loop PT transverse self-energies (Fig.1a). The explicit construction of $R_2^{ZZ}(\bar{s})$ in the $\xi_i = 1$ gauges has been given in Ref.[5]. We must now generalize this procedure to a general gauge. The chain of one-loop PT self-energies is by definition $\xi-$independent and gives a contribution proportional to

$$\left[\Pi_1^{ZZ}(s)|_{\xi_i=1} - (s - M_0^2)4g^2c_w^2 I_{WW}\right]^2 /(s-M_0^2) + \left[\Pi_1^{\gamma Z}(s)|_{\xi_i=1} - (2s - M_0^2)2g^2 s_w c_w I_{WW}\right]^2 /s \ , \tag{11}$$

where we have employed Eqs.(16d,16b) of Ref.[3]. On the other hand, using the results of Refs.[3], [4] and neglecting $\mathcal{O}(g^6)$, one finds that for $s = \bar{s}$ and general ξ_i the chain of R_ξ one-loop self-energies and pinch parts contributes

$$\begin{aligned} &\left[\Pi_1^{ZZ}(\bar{s})|_{\xi_i=1} + (\bar{s} - M_0^2)2g^2c_w^2(\xi_W - 1)\upsilon_W(\bar{s})\right]^2 /(\bar{s} - M_0^2) \\ &+ \left[\Pi_1^{\gamma Z}(s)|_{\xi_i=1} + \bar{s}g^2 s_w c_w(\xi_W - 1)\upsilon_W(\bar{s})\right]^2 /\bar{s} \\ &-4g^2c_w^2\Pi_1^{ZZ}(\bar{s})|_{\xi_i=1}\left[I_{WW}(\bar{s}) + (\xi_W - 1)\upsilon_W(\bar{s})/2\right] \qquad . \end{aligned} \tag{12}$$

The two first terms arise from the self-energy contributions in a general R_ξ gauge [4] through $\mathcal{O}(g^4)$, while the third involves the contribution of self-energies and pinch parts (Fig.1(b-d)) in the same approximation. Setting $s = \bar{s} = M_0^2 + \Pi_1^{ZZ}(\bar{s}) + ...$ in Eqs.(11,12) and subtracting the two expressions we obtain

$$\begin{aligned} R_2^{ZZ}(\bar{s}) = \ & -4g^2\left[c_w^2\Pi_1^{ZZ}(\bar{s}) + s_w c_w \Pi_1^{\gamma Z}(\bar{s})\right]|_{\xi_i=1}\left[I_{WW}(\bar{s}) + (\xi_W - 1)\upsilon_W(\bar{s})/2\right] \\ & +g^4 s_w^2 c_w^2 \bar{s}\left[4I_{WW}^2(\bar{s}) - (\xi_W - 1)^2\upsilon_W^2(\bar{s})\right] + \mathcal{O}(g^6) \ . \end{aligned} \tag{13}$$

Comparing Eq.(10) with Eq.(13) we see that the ξ_W-dependent contributions cancel in the combination $\Pi_2^{ZZ}(\bar{s}) - R_2^{ZZ}(\bar{s})$. Thus, Eq.(3) is indeed gauge-independent through $\mathcal{O}(g^4)$ if $R_2^{ZZ}(\bar{s})$ is evaluated according to the P-P method. In the neighborhood $|s - \bar{s}| \leq \mathcal{O}(g^2|\bar{s}|)$, Eq.(3) becomes

$$\begin{aligned} \widehat{\Pi}^{ZZ}(s) = \ & \Pi_1^{ZZ}(s)|_{\xi_i=1} + \Pi_2^{ZZ}(\bar{s})|_{\xi_i=1} - 4g^2c_w^2(s - \bar{s})I_{WW}(\bar{s}) \\ & +4g^2 s_w c_w \Pi_1^{\gamma Z}(\bar{s})|_{\xi_i=1} I_{WW}(\bar{s}) - 4g^4 s_w^2 c_w^2 \bar{s} I_{WW}^2(\bar{s}) + \mathcal{O}(g^6) \ . \end{aligned} \tag{14}$$

For $s = \bar{s}$, Eq.(14) reduces to Eq.(3.16) of Ref.[5]. Using Eq.(14) one finds that in the PT approach the denominator in the Z propagator can be written as

$$s - M_0^2 - \widehat{\Pi}^{ZZ}(s) - [\widehat{\Pi}^{\gamma Z}(s)]^2/s = (s - \bar{s})\left[1 - \frac{d}{ds}\widehat{\Pi}_1^{ZZ}(s)|_{s=\bar{s}}\right] + \mathcal{O}(g^6) \ , \qquad (15)$$

where it is understood that $|s - \bar{s}| \leq \mathcal{O}(g^2|\bar{s}|)$. In order to derive Eq.(15), it is convenient to add and subtract $\bar{s}$ in the l.h.s., employ $\bar{s} - M_0^2 = \Pi^{ZZ}(\bar{s}) + (\Pi^{\gamma Z}(\bar{s}))^2/\bar{s} + \ldots$, and recall the expression for $\widehat{\Pi}^{\gamma Z}$ given in Eq.(16b) of Ref[3]. Eq.(15) explicitly shows two important properties: 1) as it involves the PT self-energy $\widehat{\Pi}_1^{ZZ}(s)$, it is manifestedly ξ_i-independent 2) the zero of Eq.(15) occurs at $s = \bar{s}$, so that the pole position is not displaced. For the Z case, the latter property was already derived in the particular case of the $\xi_i = 1$ gauges [5]. As explained in Refs.[5], [6], using the scaling approximation for $\mathcal{I}m\widehat{\Pi}_1^{ZZ}(s)$ one can transform Eq.(15) into the characteristic $s-$dependent Breit-Wigner resonance employed in the LEP analysis, so that the connection with the LEP observables becomes explicit.

One can readily carry out the same analysis for the W self-energy . In this case there are no mixing complications but the R_ξ gauge dependence is governed by three parameters ξ_i $(i = W, Z, \gamma)$. One finds

$$\Pi_2^{WW}(\bar{s}) = \Pi_2^{WW}(\bar{s})|_{\xi_i=1} - g^2\Pi_1^{WW}(\bar{s})|_{\xi_i=1}F(\xi_i, \bar{s}) + \mathcal{O}(g^6) \ , \qquad (16)$$

$$F(\xi_i, \bar{s}) = c_w^2[(\xi_W - 1)\upsilon_{WZ}(\bar{s}) + (W \leftrightarrow Z)] + s_w^2[(\xi_W - 1)\upsilon_{W\gamma}(\bar{s}) + (W \leftrightarrow \gamma)] \ , \qquad (17)$$

where $(i \leftrightarrow j)$ is obtained from the preceding term by interchanging the indices in $(\xi_i - 1)\upsilon_{ij}(\bar{s})$. The gauge dependence is contained in $F(\xi_i, \bar{s})$. Following the P-P method we obtain

$$R_2^{WW}(\bar{s}) = -4g^2\Pi_1^{WW}(\bar{s})|_{\xi_i=1}\left[c_w^2 I_{ZW}(\bar{s}) + s_w^2 I_{\gamma W}(\bar{s})\right] - g^2\Pi_1^{WW}(\bar{s})|_{\xi_i=1}F(\xi_i, \bar{s}) \ . \qquad (18)$$

Again $\Pi_2^{WW}(\bar{s}) - R_2^{WW}(\bar{s})$ is ξ_i-independent and in the interval $|s - \bar{s}| \leq \mathcal{O}(g^2|\bar{s}|)$ we find

$$\widehat{\Pi}^{WW}(s) = \Pi_1^{WW}(s)|_{\xi_i=1} + \Pi_2^{WW}(\bar{s})|_{\xi_i=1} - 4g^2(s - \bar{s})\left[c_w^2 I_{ZW}(\bar{s}) + s_w^2 I_{\gamma W}(\bar{s})\right] + \mathcal{O}(g^6) \ . \quad (19)$$

Alternatively, the ξ_i-independence of $\widehat{\Pi}_2^{WW}(\bar{s})$ can be derived by directly evaluating $\widehat{\Pi}_1^{WW}(\bar{s}) + \widehat{\Pi}_2^{WW}(\bar{s}) - \Pi_1^{WW}(\bar{s}) - \Pi_2^{WW}(\bar{s})$ in a general ξ_i gauge [2]. Using Eq.(19) the

propagator's denominator becomes

$$s-(M_0^W)^2-\widehat{\Pi}^{WW}(s)=(s-\bar{s})\left[1-\frac{d}{ds}\widehat{\Pi}_1^{WW}(s)|_{s=\bar{s}}\right]+\mathcal{O}(g^6)\ , \tag{20}$$

in analogy with the Z case.

In summary, by restricting s to the neighborhood $|s-\bar{s}|\leq\mathcal{O}(g^2|\bar{s}|)$ of the propagator's pole one can test the gauge dependence of the ZZ and WW self-energies through $\mathcal{O}(g^4)$. In both cases we find that the P-P method leads to gauge independent amplitudes. Because of our restriction to the resonance region, and our neglect of $\mathcal{O}(g^6)$ terms, this test amounts to a necessary rather than a sufficient condition. On the other hand, it is important to emphasize that this domain is of special physical significance.

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Figure Caption

Chain of one-loop transverse PT self-energies through $\mathcal{O}(g^4)$ and a class of related pinch parts.

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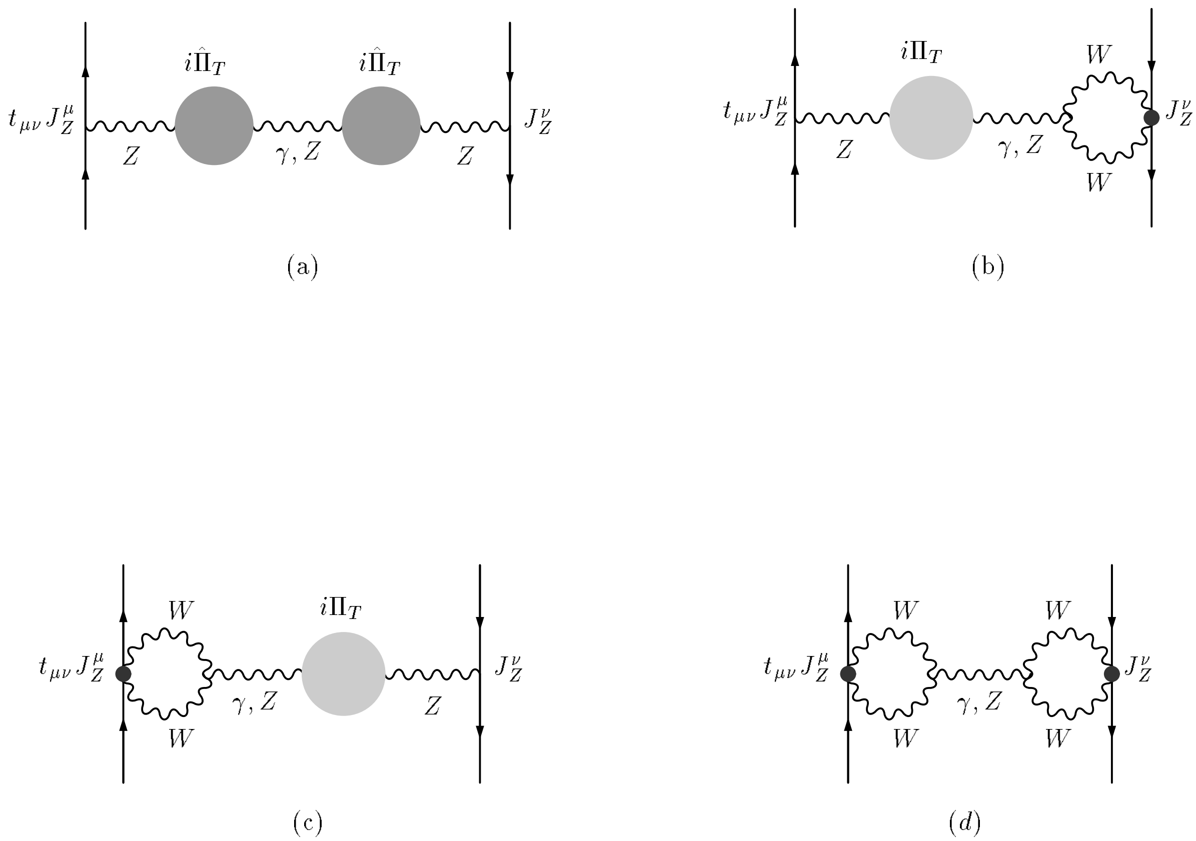

$t_{\mu\nu} J_Z^\mu$
$i\hat{\Pi}_T$
$i\hat{\Pi}_T$
J_Z^ν
Z
γ, Z
Z
(a)
$t_{\mu\nu} J_Z^\mu$
$i\Pi_T$
W
J_Z^ν
Z
γ, Z
W
(b)
$t_{\mu\nu} J_Z^\mu$
W
$i\Pi_T$
J_Z^ν
W
γ, Z
Z
(c)
$t_{\mu\nu} J_Z^\mu$
W
W
J_Z^ν
W
γ, Z
W
(d)

Figure 1